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Tobin's *Q*, Debt Overhang, and Investment

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ABSTRACT

Incorporating debt in a dynamic real options framework, we show that underinvestment stems from truncation of equity's horizon at default. Debt overhang distorts both the level and composition of investment, with underinvestment being more severe for long-lived assets. An empirical proxy for the shadow price of capital to equity is derived. Use of this proxy yields a structural test for debt overhang and its mitigation through issuance of additional secured debt. Using measurement error-consistent GMM estimators, we find a statistically significant debt overhang effect regardless of firms' ability to issue additional secured debt.

IN DYNAMIC INVESTMENT MODELS, the shadow price of capital, or marginal q, is a sufficient statistic for investment.[1] Since marginal q is unobservable, Tobin's average Q, the ratio of equity plus debt value to replacement cost of the capital stock, is commonly used as an empirical proxy. Hayashi (1982) and Abel and Eberly (1994) provide formal justifications for this practice, deriving conditions under which average Q and marginal q are equal. A drawback of both models is that they preclude any role for financial structure by assuming that the firm is financed exclusively with equity.

Starting with the Abel and Eberly (1994) model of first-best investment, where marginal and average Q are equal, this paper analyzes the investment policy of an equity-maximizing firm with long-term debt outstanding.[2] In this situation, levered equity's marginal q does not reflect the value of post-default investment returns accruing to existing lenders. However, such returns are capitalized into the price of existing debt and thus included in the numerator of average Q. This implies that marginal q is less than average Q for firms with long-term debt. We show that levered marginal q is equal to average Q minus the capital-normalized value of existing lenders' claim to recoveries in default.

A second expression relating levered and unlevered marginal q is also derived, facilitating analysis of the role played by asset life. The importance of

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[1] See Mussa (1977) and Abel (1983) for early treatments.

[2] This is the objective function in Myers (1977). Recent numerical work includes Mello and Parsons (1992); Mauer and Triantis (1994); Parrino and Weisbach (1999); Morellec (2001); Moyen (2001); and Titman and Tsyplakov (2002).

asset life has thus far been neglected in the literature on agency costs of debt. We show that levered marginal q equals unlevered marginal q minus the shadow price of undepreciated capital at the time of default. For a firm investing in multiple capital goods, the percentage of undepreciated capital remaining at the time of default is higher for physical assets with low rates of depreciation. This implies that overhang distorts the composition as well as the level of investment, with the underinvestment problem being more severe for long-lived assets.

In order to test the theory, an empirical proxy for the overhang correction term is computed as the capital-normalized value of *total* lender recoveries in default. In the model, the optimal investment policy is shown to be linear in average Q and the overhang correction term, with the latter predicted to have a negative coefficient. Under the alternative hypothesis of first-best investment, the correction term has a coefficient of zero. Using the measurement error-consistent generalized method of moments (GMMs) estimators of Erickson and Whited (2002), we find that the overhang correction term is always statistically significant.

The model also predicts that the absolute value of the coefficient on the overhang correction term will be smaller for firms that issue additional secured debt in order to reduce the spillover benefit that new investment provides to existing lenders. To test this hypothesis, we split the sample into two groups, based on two alternative indicators for superior ability to issue new secured debt: high bond ratings or high industry-specific recovery ratios. Regardless of the sorting criterion, we fail to reject the null hypothesis that the coefficient on the overhang correction does not depend on firm status.

The economic significance of the debt overhang effect varies systematically with debt ratings. The term subtracted from average Q, the normalized value of total default recoveries, is much larger for low-rated firms. For instance, the mean ratio of the overhang correction term to average Q is 11% for firms below investment grade and 0.8% for those above investment grade. Therefore, while we find that the statistical significance of the overhang effect is not mitigated by the issuance of future secured debt, the economic significance of the overhang channel is relatively small for healthy firms.

In a related paper, Whited (1992) finds that an Euler equation without credit constraints performs well for nondistressed firms, but is rejected for distressed firms. The addition of a credit constraint improves the performance of the Euler equation when applied to distressed firms. In our model, overhang and credit constraints are linked, since overhang is most severe for firms that are unable to issue additional secured debt. Whited's finding of significant departures from first-best for distressed firms is consistent with our empirical results, since distressed firms exhibit the largest wedge between levered equity's marginal q and average Q, with the latter being a sufficient statistic for investment under first-best.

Lang, Ofek, and Stulz (1996) use average Q as a proxy for growth opportunities, with book leverage included as a regressor. Book leverage enters significantly only for low-Q firms. Although our evidence is consistent with their findings, the mapping between their tests and the underlying theory is unclear since the leverage ratio is an imperfect proxy for the overhang correction. In addition, it is impossible to determine whether the magnitude of the estimated coefficient is consistent with the overhang hypothesis. Using a structural approach, we find that the negative effect of debt on investment is actually somewhat stronger than that implied by the overhang channel working in isolation.

The remainder of the paper is organized as follows. Section I presents the basic model assuming that the senior debt contract prohibits issuance of additional secured debt. Section II extends the basic model, allowing for issuance of additional secured debt. Section III includes the empirical tests.

I. Basic Model

A. Contracting Framework

In the interest of transparency, we build directly upon the real options framework of Abel and Eberly (1994) and adopt their notation where possible. In contrast to their unlevered firm, consider a firm that has issued senior long-term debt with a promise to pay an infinite stream of coupon payments, b. The assumption of a consol bond is motivated by our desire to model the agency costs of debt, since debt maturing before a particular growth option becomes available is irrelevant to that investment decision. The optimal debt commitment (b) is not derived since the objective of this paper is to determine the empirical implications of overhang for structural investment equations. The derived structural investment equation holds for arbitrary b.

The firm has the option to default given limited liability, and there are no deviations from absolute priority. Section I assumes that senior debt contains a covenant specifying that future debt issuance must be subordinate instantaneous debt that does not affect the recovery claim of the senior lender. This assumption is made for two reasons. First, as suggested by Myers (1977), eroding senior debt value with additional debt issuance is a device for mitigating underinvestment. Second, provisions limiting future debt issuance are commonly included in debt covenants. Smith and Warner (1979) find that 90.8% of their sampled covenants contain some restriction on future debt issuance. The extended model presented in Section II allows a portion of incremental investment to be financed with new secured debt.

The investment program maximizes equity value, with contractual commitment to first-best ruled out. This is a reasonable assumption, given the difficulty courts would have in determining whether particular investments are positive NPV. Renegotiation of debt is also ruled out. With widely dispersed creditors, the costs of renegotiation are prohibitive. Further, as discussed by Smith and Warner (1979), public debt is subject to the Trust Indenture Act of 1939, which places stringent conditions on restructurings. An interesting direction for future research, not explored here, is whether firms relying on bank or privately placed debt avoid the overhang problem through renegotiation.

Internal cash is the first source of funds, with additional investment financed with capital infusions. As an abstraction, it is helpful to view the manager as sole shareholder, with sufficient wealth to fund desired investment if it exceeds internal cash. The contributions may be considered to be equity or simply viewed as the manager lending to himself. The two are economically equivalent. We adopt the labeling convention of treating the manager's value function as equity.

B. Investment Problem

The firm is a price-taker in both output and input markets. Let π represent maximized instantaneous operating profits, which is a function of the capital stock K and state ε. This specification allows for price, wage, and productivity shocks. Letting F represent the production function, p the output price, w the wage rate, and N variable labor inputs, maximized operating profits is

$$\pi(K, \varepsilon) \equiv \max_N p(\varepsilon)F(K, N, \varepsilon) - w(\varepsilon)N. \tag{1}$$

The evolution of (K, ε) is governed by

$$dK_t = (I_t - \delta K_t)\,dt, \tag{2}$$

$$d\varepsilon_t = \mu(\varepsilon_t)\,dt + \sigma(\varepsilon_t)\,dW_t. \tag{3}$$

The variables I and δ represent investment and the depreciation rate, respectively. Here W is a standard Wiener process, with the drift and volatility for ε satisfying the necessary conditions for the existence of a unique solution to the stochastic differential equation.[3] The diffusion process for the state variable is sufficiently general to allow for competitive dynamics that may affect the path of ε.

Let c represent the total cost of changing the capital stock of the firm, with c being a function of (I, K)

$$c(I, K) = P(I)I + G(I, K) + \Phi(I)a(K). \tag{4}$$

The first term represents direct costs of investment, with the price at which capital is purchased (P^+) possibly exceeding that at which it is sold (P^-). The function G represents the cost of adjusting plant and equipment and is smooth, increasing in I on $\Re_+$, decreasing in I on $\Re_-$, decreasing in K, and strictly convex in both arguments. The function Φ is an indicator for nonzero investment, with $a(K)$ representing fixed costs.

Smith and Warner (1979) find that 35.6% of their sample of debt covenants place limits on the disposition of assets. In the model

$$\begin{aligned} \text{no covenant restricting asset sales} &\Rightarrow I \in \Delta \equiv \Re, \\ \text{covenant restricting asset sales} &\Rightarrow I \in \Delta \equiv \Re_+. \end{aligned} \tag{5}$$

[3] We assume that stochastic integrals with respect to W are martingales. See Duffie (1996) and Karatzas and Shreve (1988) for regularity conditions.

Let date zero represent the present for simplicity and T the stochastic default time. One should think of the default threshold as being a contour in (K, ε) space. Note that, like I, the stopping time T is not chosen at date zero, but is chosen optimally based on current information.[4] Letting S denote the value of equity (stock), we have

$$S(K_0, \varepsilon_0) \equiv \max_{I \in \Delta, T} E\left[\int_0^T e^{-rt}[\pi(K_t, \varepsilon_t) - c(I_t, K_t) - b]\,dt\right]. \tag{6}$$

The Bellman equation for this problem is[5]

$$rS(K, \varepsilon) = \max_{I \in \Delta} \pi(K, \varepsilon) - c(I, K) - b + \frac{1}{dt}E[dS]. \tag{7}$$

Applying Ito's lemma, the Bellman equation may be written as

$$\begin{aligned} rS(K, \varepsilon) = \max_{I \in \Delta} \pi(K, \varepsilon) - c(I, K) - b + (I - \delta K)S_K \\ + \mu(\varepsilon)S_\varepsilon + \tfrac{1}{2}\sigma^2(\varepsilon)S_{\varepsilon\varepsilon}. \end{aligned} \tag{8}$$

The key variable determining investment is the shadow price of capital to equity, which is denoted q. More formally

$$q(K, \varepsilon) \equiv S_K(K, \varepsilon). \tag{9}$$

Ignoring terms on the right side of the Bellman equation not involving I, the manager's instantaneous problem is

$$\max_{I \in \Delta} Iq(K, \varepsilon) - c(I, K). \tag{10}$$

Assume first that investment is perfectly reversible ($P^+ = P^-$), with no fixed costs of adjustment ($a = 0$), and no covenant restriction on asset sales. Under these three assumptions, the function c is smooth and optimal instantaneous investment equates the marginal cost of investment with marginal q

$$c_I(I^*, K) = q(K, \varepsilon). \tag{11}$$

Abel and Eberly's (1994) characterization of the investment rule in the presence of fixed costs of adjustment and irreversibilities carries over to our setting, with the exception that the relevant shadow price in our model is the shadow price of capital to *equity*. In the interest of avoiding redundancy, we refer readers to their paper, noting that fixed costs of adjustment and irreversibilities generate a region where optimal investment is zero and insensitive to perturbations in the shadow price of capital. It is sufficient to note that the optimal investment rule for the basic model presented in this section satisfies

$$I^* c_I(I^*, K) = I^* q(K, \varepsilon). \tag{12}$$

[4] Formally, investment and stopping time are adapted to the natural filtration.
[5] Verification is standard (see Fleming and Soner (1993)).

The optimality condition (12) also holds if senior debt includes a covenant prohibiting asset sales. In this case, when the shadow price of capital is sufficiently low, the nonnegativity constraint binds and $I^* = 0$.

C. Derivation of Marginal q

Proposition 1 contains a simple expression for q that is useful in interpreting the nature of the underinvestment problem.

PROPOSITION 1: *The shadow price of capital to equity is*

$$q(K_0, \varepsilon_0) = E\left[\int_0^T e^{-(r+\delta)t}[\pi_K(K_t, \varepsilon_t) - c_K(I_t, K_t)]\,dt\right].$$

Proof: See Appendix A.

Marginal q is simply the discounted value of the effect of a unit of installed capital on cash flow to equity prior to the expected default date. The discount rate is $r + \delta$ due to exponential depreciation. Lemma 1 in Abel and Eberly (1994) is the analog under first-best. The only difference between their expression for the shadow price of capital to the unlevered firm and that in Proposition 1 is that their upper limit of integration is infinity. Therefore, underinvestment relative to first-best is due to truncation of equity's horizon.

It is generally argued that the underinvestment problem is more severe for firms in distress. In the model, distress corresponds to short horizons (low T in expectation). More formally, underinvestment is more severe in distress due to the smooth pasting condition ($q = 0$) that is satisfied along the optimal default contour.[6] For reasonable specifications of the adjustment cost function, the optimal investment rule when q approaches zero entails negative investment if allowed. Here one sees an obvious role for debt covenants.[7]

Note that underinvestment is properly treated as distinct from the asset substitution problem, first discussed in Jensen and Meckling (1976). It is possible to extend the model and permit endogenous risk choice by including σ as a control. The Bellman equation (8) is separable in (I, σ), so that the characterization of the optimal investment level remains correct when σ is a control. High volatility is optimal when equity is convex in ε, which is necessarily the case near the default contour, indicating that the optimal policy for distressed firms entails paying the maximum dividend allowed by the debt covenant, since q approaches zero, and speculating with any remaining funds.

In the event of default, the lender(s) take over an unlevered firm and implement first-best. The value function for the unlevered firm is denoted V. Following Leland (1994), it is assumed that a fraction $\alpha \in [0, 1]$ is lost during the reorganization process. Absent issuance of additional collateralized debt,

[6] See Dumas (1991) for a discussion of smooth pasting conditions.

[7] See Morellec (2001) for analysis of the relation between covenants, liquidity in secondary asset markets, and underinvestment.

the value of the senior debt, denoted D, is equal to the value of coupon payments plus total recoveries in default:

$$D(K_0, \varepsilon_0) = E\left[\int_0^T e^{-rt} b\, dt + e^{-rT}(1-\alpha)V(K_T, \varepsilon_T)\right]. \tag{13}$$

It is convenient to split the senior debt value into two pieces, the value of the coupons and the value of total recoveries in default (R):

$$D(K_0, \varepsilon_0) = E\left[\int_0^T e^{-rt} b\, dt\right] + R(K_0, \varepsilon_0), \tag{14}$$

$$R(K_0, \varepsilon_0) \equiv (1-\alpha)E[e^{-rT} V(K_T, \varepsilon_T)]. \tag{15}$$

Average Q is equal to the ratio of the market value of the firm to the current capital stock:

$$Q(K_0, \varepsilon_0) \equiv \frac{S(K_0, \varepsilon_0) + D(K_0, \varepsilon_0)}{K_0}. \tag{16}$$

The relationship between average Q and equity's marginal q is given in Proposition 2.[8]

PROPOSITION 2: *If π is homogeneous degree one in K; c is homogeneous degree one in (I, K); and the firm is prohibited from issuing additional secured debt, then the shadow price of capital to equity is equal to average Q, less the normalized current market value of total default recoveries (R):*

$$q(K_0, \varepsilon_0) = Q(K_0, \varepsilon_0) - \frac{R(K_0, \varepsilon_0)}{K_0}.$$

Proof: See Appendix A.

Note that in the absence of debt, we arrive at Lemma 2 from Abel and Eberly (1994), which states that for the unlevered firm $q = Q$ under linear homogeneity. The intuition behind Proposition 2 is as follows. The manager of the levered firm takes into account only those benefits accruing prior to default. Average Q overstates marginal q by incorporating post-default returns to investment through its inclusion of the portion of senior debt value attributable to recoveries in default. Post-default investment returns must be netted out in order to determine the shadow price of capital to equity.

Corollary 1, which might initially seem counterintuitive, follows directly from the preceding argument.

COROLLARY 1: *If the firm is prohibited from issuing additional secured debt, then for a given coupon on existing (sunk) debt, larger bankruptcy costs imply a smaller wedge between average $Q(K, \varepsilon)$ and levered marginal $q(K, \varepsilon)$. When all value is lost in bankruptcy ($\alpha = 1$), $Q(K, \varepsilon) = q(k, \varepsilon)$.*

[8] If π and c are homogeneous degree ρ, then $q = \rho(Q - R/K)$. Although treated in Abel and Eberly (1994), the assumption seems dubious for $\rho \neq 1$.

Proof: Consider two firms indexed by h and l with $\alpha_h > \alpha_l$ at the same point in (K, ε) space with the same coupon commitment b. The S and q functions are identical, as is the stopping policy T. Therefore, $R_h(K, \varepsilon) < R_l(K, \varepsilon)$. If $\alpha_h = 1$, then $R_h(K, \varepsilon) = 0$. The result follows from Proposition 2. Q.E.D.

The intuition for Corollary 1 is as follows. The wedge between average Q and marginal q stems from value left on the table for the senior lender. When bankruptcy costs are high, recoveries in default are low, resulting in a smaller wedge. Note that Corollary 1 is based on the assumption that the benefits from issuing the senior debt are already sunk. Section II considers an alternative setting in which the firm is able to issue new secured debt in order to finance a portion of incremental investment.

Drawing inferences about investment incentives from average Q is particularly misleading near the default date. This is because the smooth-pasting condition for endogenous default demands $q(K_T, \varepsilon_T) = 0$. However,

$$Q(K_T, \varepsilon_T) = \frac{D(K_T, \varepsilon_T)}{K_T} = \frac{(1-\alpha)V(K_T, \varepsilon_T)}{K_T}, \tag{17}$$

which is strictly positive if $\alpha < 1$. Intuitively, investment near default produces benefits accruing almost entirely to lenders, which are capitalized into debt value and average Q.

D. Role of Asset Life

Given that overhang is due to truncation of equity's horizon, intuition suggests that the underinvestment problem is more severe for long-lived assets. We confirm this intuition by contrasting the investment decisions of levered and unlevered firms. Consider a price-taking firm with Cobb–Douglas production technology under constant returns to scale. The price process is stochastic and adjustment costs are independent of K.

Summarizing the assumptions:

$$\begin{aligned} F(K, N) &= N^{\theta} K^{1-\theta}, \\ \theta &\in (0, 1), \\ \pi(K, p) &\equiv \max_N pF(K, N) - wN, \\ dp_t &= \sigma p_t \, dW_t, \\ c_K(I, K) &\equiv 0 \quad \text{for all } (I, K). \end{aligned} \tag{18}$$

Under these assumptions, the profit function is linear in K. In particular,[9]

$$\pi(K, p) = hp^{\zeta} K, \quad h \equiv (1-\theta)\theta^{\theta/(1-\theta)} w^{-\theta/(1-\theta)} > 0, \quad \zeta \equiv 1/(1-\theta) > 1. \tag{19}$$

[9] See Abel and Eberly (1994, p. 1378) for a derivation.

We now relate the shadow price of capital to the levered firm (q_l), with that of the unlevered firm (q_u). Abel and Eberly (1994) show that

$$q_u(p_t) = h\int_0^\infty e^{-(r+\delta)s}E_t\left[p_{t+s}^\zeta\right]ds = \frac{hp_t^\zeta}{r+\delta-\frac{1}{2}\zeta(\zeta-1)\sigma^2}. \tag{20}$$

Applying Proposition 1, it follows that

$$q_l(p_0) = E\left[\int_0^T e^{-(r+\delta)t}hp_t^\zeta\,dt\right]. \tag{21}$$

Combining (20) and (21), it follows that

$$q_l(p_0) = q_u(p_0) - E\left[e^{-rT}(e^{-\delta T})q_u(p_T)\right]. \tag{22}$$

Expression (22) clarifies the source of underinvestment. The equity-maximizing manager does not incorporate post-default returns into his computation of the shadow price. At the default date T, the value to the unlevered firm of the undepreciated portion of a machine purchased at date zero is equal to $e^{-\delta T}q_u(p_T)$. As shown in (22), it is this term that creates underinvestment relative to first-best. Note that in addition to the expected horizon (T), the depreciation rate (δ) is an important determinant of the difference between the shadow price of capital for unlevered and levered firms.

II. Extended Model with Secured Debt

Myers (1977) noted that firms can mitigate the overhang problem if incremental investment is (partially) financed with new secured debt. In our neoclassical capital accumulation framework, issuance of new secured debt mitigates overhang by allowing equity to capture a portion of post-default returns by pledging collateral to new lenders.[10] After modeling this effect, the objective is to determine how secured loans affect the relationship between marginal q and average Q.

Assume that for each new unit of capital purchased, the senior debt covenant allows the firm to raise a fixed amount λ through zero coupon secured debt. The parameter λ is interpreted as a measure of the firm's ability to issue new secured debt. In the model, the entire value of new secured debt is derived from the underlying pledged collateral. We assume that the new debt carries no coupon payments in order to highlight the role of pledged collateral. In particular, the value of pledged collateral accounts for the entire increase in NPV to equity, constituting a transfer from senior lenders.

Each new secured loan gives the respective lender rights to some fraction (γ_t) of the particular piece of capital that her funds partially financed. Senior debt retains rights over existing capital and the unsecured portion $(1-\gamma_t)$ of new

[10] Deviations from absolute priority in favor of equity would serve a similar purpose.

capital. Since all debt is secured by some asset, disinvestment is prohibited. In the event of default, total reorganized firm value is allocated in proportion to the percentage of physical capital over which individual lenders have rights. For instance, if a particular lender has rights over a fraction γ_t of a machine purchased at time t and default occurs at time $T > t$, then the value recovered by this lender is

$$\gamma_t \left(\frac{e^{-\delta(T-t)}}{K_T} \right) (1-\alpha) V(K_T, \varepsilon_T). \tag{23}$$

Lemma 1, proved by Abel and Eberly (1994), is a useful result utilized below.

LEMMA 1: *If π is homogeneous degree one in K and c is homogeneous degree one in (I, K), then unlevered firm value is linear in K and may be represented as*

$$V(K, \varepsilon) = v(\varepsilon) K.$$

Using Lemma 1, the expression given in (23) simplifies to

$$\gamma_t (1-\alpha) e^{-\delta(T-t)} v(\varepsilon_T). \tag{24}$$

Since secured debt is priced at λ, it must be the case that for all $t < T$:

$$\lambda = \gamma_t (1-\alpha) E_t [e^{-(r+\delta)(T-t)} v(\varepsilon_T)]. \tag{25}$$

The pricing identity (25) is utilized in the derivation of marginal q presented in Appendix A.

The equity value function in the extended model is

$$S(K_0, \varepsilon_0) \equiv \max_{I \in \Delta, T} E\left[\int_0^T e^{-rt} [\pi(K_t, \varepsilon_t) - c(I_t, K_t) + \lambda I_t - b] \, dt \right]. \tag{26}$$

Note that the only change in the objective function is the term λI_t, which represents the amount raised with new secured debt. Following the same steps as in Section I, it follows that the optimal investment policy satisfies

$$I^* \left[c_I(I^*, K) - \lambda \right] = I^* q(K, \varepsilon). \tag{27}$$

Outside of corner solutions, at which $I^* = 0$, the optimality condition is

$$c_I(I^*, K) = q(K, \varepsilon) + \lambda. \tag{28}$$

Proposition 3 is useful in interpreting condition (28).

PROPOSITION 3: *In the extended model with the firm being allowed to issue additional secured debt, the shadow price of capital to equity is*

$$q(K_0, \varepsilon_0) = E\left[\int_0^T e^{-(r+\delta)t} [\pi_K(K_t, \varepsilon_t) - c_K(I_t, K_t)] \, dt \right].$$

Proof: See Appendix A.

Note that the shadow price of capital to equity is the same as that presented in Section I. This is not surprising, given that the secured debt does not change the cash flow rights of equity, nor the value of its claim in default, which is zero. Rather, the secured debt has simply redistributed some of the recovery claim value to new lenders. This transfer is captured by equity when new debt is issued. Condition (28) shows that the ability to collateralize new debt serves as a subsidy to investment. The effect is equivalent to reducing the purchase price of capital from P^+ to $P^+ - \lambda$, with the optimality condition indicating that investment is strictly increasing in λ, excluding corner solutions.

Despite the fact that the shadow price of capital to equity is unchanged, Proposition 4 demonstrates that the mapping between marginal q and average Q *is* changed when the firm is allowed to issue new secured debt.

PROPOSITION 4: *If π is homogeneous degree one in K; c is homogeneous degree one in (I, K); and the firm raises λ per unit of new capital with pledged collateral, then the shadow price of capital to equity is equal to average Q, less the normalized current market value of total default recoveries, plus the normalized value of future pledged collateral:*

$$q(K_0, \varepsilon_0) = Q(K_0, \varepsilon_0) - \frac{R(K_0, \varepsilon_0)}{K_0} + \frac{\lambda E\left[\int_0^T e^{-rt} I_t \, dt\right]}{K_0}.$$

Proof: See Appendix A.

Proposition 2 assumes that senior debt recovers all value in default, implying that the entire normalized recovery claim value (R/K) must be subtracted from Q in order to compute equity's marginal q. In contrast, when the firm is able to issue new secured debt, the value of existing debt includes R minus the value pledged to new lenders. In order to derive equity's marginal q, only this net value must be subtracted from average Q. To illustrate this more formally, note that the value of senior debt in the context of the extended model is

$$D(K_0, \varepsilon_0) = E\left[\int_0^T e^{-rt} b \, dt\right] + R(K_0, \varepsilon_0) - \lambda E_0\left[\int_0^T e^{-rt} I_t \, dt\right]. \quad (29)$$

Comparison of (14) and (29) reveals that in both the basic and extended models, the wedge between average Q and levered marginal q is attributable to the portion of senior debt value coming from recoveries in default.

III. Empirical Testing

A. Structural Investment Equation

Following a number of authors, empirical estimation is based upon the following linear homogeneous quadratic adjustment cost function:[11]

$$c(I_{jt}, K_{jt}) = I_{jt} + \frac{1}{2}\eta K_{jt}\left[\left(\frac{I}{K}\right)_{jt} - \delta\right]^2. \quad (30)$$

[11] See Summers (1981), Poterba and Summers (1983), Chirinko (1987), and Whited (1992), for example.

Under this adjustment cost function, the optimality condition from the extended model (28) simplifies to

$$\left(\frac{I}{K}\right)_{jt} = \left(\delta - \frac{1}{\eta}\right) + \frac{\lambda_j}{\eta} + \frac{1}{\eta}q_{jt}. \tag{31}$$

Although we assume a smooth adjustment cost function for the purpose of estimation, the derivations of q presented above allow for a wedge between the buy and sell prices of capital, fixed costs of adjustment, and prohibitions on asset sales. Data limitations preclude estimation of the nonlinear investment-q relationships implied by these frictions. See Abel and Eberly (2001) and Barnett and Sakellaris (1998) for empirical tests of the theory under the assumption that the manager implements first-best.

Estimation is based on the optimality condition (31). Substituting in the q expression from Proposition 4 yields

$$\left(\frac{I}{K}\right)_{jt} = \left(\delta - \frac{1}{\eta}\right) + \frac{1}{\eta}Q_{jt} + \frac{1}{\eta}\lambda_j - \frac{1}{\eta}\left(\frac{R}{K}\right)_{jt} + \frac{1}{\eta}\left(\frac{\lambda_j E_t\left[\int_t^T e^{-r(s-t)} I_s\, ds\right]}{K_t}\right) + u_{jt}. \tag{32}$$

Now define the parameter Λ_{jt} as follows:

$$\Lambda_{jt} \equiv \frac{\lambda_j E_t\left[\int_t^T e^{-r(s-t)} I_s\, ds\right]}{R_{jt}} \in [0, 1].$$

Therefore, Λ_{jt} is a measure of the relative importance of collateral pledged to future lenders. For instance, if the firm cannot secure any future loans, $\Lambda_{jt} = 0$. Conversely, if future secured lenders capture all post-default returns, then $\Lambda_{jt} = 1$. Using this definition, we may rewrite the regression equation (32) as

$$\left(\frac{I}{K}\right)_{jt} = \left(\delta - \frac{1}{\eta}\right) + \frac{1}{\eta}\lambda_j + \frac{1}{\eta}Q_{jt} - \frac{1}{\eta}\left(\frac{R}{K}\right)_{jt} + \frac{\Lambda_{jt}}{\eta}\left(\frac{R}{K}\right)_{jt} + u_{jt}. \tag{33}$$

Suppose firms can be classified into “high” and “low” groups according to their ability to collateralize future loans, with $\Lambda_H > \Lambda_L$ and $\lambda_H > \lambda_L$. Sorting in this way and letting *HIGH* be an indicator function, the regression equation (33) becomes:

$$\left(\frac{I}{K}\right)_{jt} = \left(\delta - \frac{1}{\eta} + \frac{\lambda_L}{\eta}\right) + \frac{1}{\eta}Q_{jt} + \left(\frac{\lambda_H - \lambda_L}{\eta}\right) HIGH_{jt} - \left(\frac{1-\Lambda_L}{\eta}\right)\left(\frac{R}{K}\right)_{jt} + \left(\frac{\Lambda_H - \Lambda_L}{\eta}\right)\left(HIGH \times \frac{R}{K}\right)_{jt} + u_{jt}. \tag{34}$$

Equation (34) serves as the theoretical basis for the empirical specification. The sorting of firm-years into high and low categories is based on two measures of ability to issue future secured debt. The first sorting criterion is whether the S&P debt rating is at/below BB+ or above BB+, which is the traditional cutoff for determining "investment grade" status. Firms with debt obligations below investment grade are more likely to face binding debt covenant prohibitions on future debt issuance. The second sorting criterion is whether the firm belongs to an industry that has historically enjoyed a recovery ratio on defaulted debt above the median value in our sample. Recovery ratios by three-digit SIC code are taken from Altman and Kishore (1996), who compute averages over the period 1971–1995. Firms in industries with recovery ratios at or above the sample median of 41.5% are categorized as having *HIGH* status.

B. Estimation Procedure

Caballero (1997) documents poor performance of the standard Q theory in explaining aggregate and firm-level investment. Frequently, the coefficients on Q are insignificant, while measures of liquidity enter positively and significantly. A possible explanation is that average Q is necessarily measured with error. Using measurement error-consistent GMM estimators, Erickson and Whited (2001) find that most common approaches generate poor proxies for true Q. Therefore, in moving from theory to data, our major concern is dealing with the lack of a clean measure of the shadow price of capital to the firm, which is equal to average Q under the stated homogeneity assumptions.

For the purpose of empirical testing, we depart from ordinary least squares (OLS) and employ the measurement error-consistent GMM estimator presented in Erickson and Whited (2002). Using this estimator, Erickson and Whited (2000) find that many stylized facts in the empirical investment literature potentially result from measurement error in Q. In particular, they find that cash flow becomes insignificant, while the point estimates of the Q-coefficient roughly triple in magnitude relative to the OLS baseline estimation. We describe how their GMM estimator is applied to our model below.

Let $\mathbf{z}$ be the vector of explanatory variables, excluding Q. In addition to the explanatory variables generated directly by the theory (see (34)), we estimate separate cash flow coefficients for firms with debt above and below investment grade. Letting CF_A and CF_B denote cash flow for firms above and below investment grade, respectively, we have:

$$\mathbf{z} \equiv \left\{ HIGH, \frac{R}{K}, \left(HIGH \times \frac{R}{K} \right), CF_A, CF_B \right\}. \tag{35}$$

There are two motivations for adding cash flow in this way. First, some would argue that liquidity is an important determinant of investment, with the effect being most pronounced for weak firms. Second, adding these regressors was necessary to ensure a reasonable degree of confidence in satisfying the identifying assumptions required to use the GMM estimators.[12]

[12] Interacting cash flow and a size dummy, as in Erickson and Whited (2000), produced weak evidence for model identification.

Let $\mathbf{B}$ represent the vector of regression coefficients on $\mathbf{z}$ and β the true coefficient on unobserved Q, which, according to the model, is the reciprocal of η. Then the regression equation (34) for firm j may be written as

$$\left(\frac{I}{K}\right)_j \equiv i_j = \mathbf{z}_j\mathbf{B} + \beta Q_j + u_j. \tag{36}$$

The true value of Q is unobserved, with x denoting the noisy proxy, which is represented as

$$x_j = \text{constant} + Q_j + \epsilon_j. \tag{37}$$

Now let ω denote the residual from the projection of Q on $\mathbf{z}$:

$$\omega_j \equiv Q_j - \mathbf{z}_j\mathbf{m}_Q, \quad \mathbf{m}_Q \equiv [E(\mathbf{z}'_j\mathbf{z}_j)]^{-1}E[\mathbf{z}'_jQ_j]. \tag{38}$$

The following assumptions are made:

$$\begin{aligned}
&\text{A1}: \quad E[u_j] = 0, \text{ independent of } (\mathbf{z}_j, Q_j).\\
&\text{A2}: \quad (u_j, \epsilon_j, \mathbf{z}_j, Q_j) \sim \text{i.i.d. for all } j.\\
&\text{A3}: \quad E[\epsilon_j] = 0, \text{ independent of } (u_j, \mathbf{z}_j, Q_j).\\
&\text{A4}: \quad \beta \neq 0.\\
&\text{A5}: \quad E\left[\omega_j^3\right] \neq 0.
\end{aligned}$$

A1 and A2 are standard. Given the numerous approximations that are necessarily made in constructing any measure of Q, it is important to note that A3 is satisfied for classical measurement error, implying the GMM estimator is robust, while the presence of such measurement error causes OLS to produce downward-biased Q coefficients and contaminated coefficients on other regressors.[13] A test for satisfaction of the identifying conditions A4 and A5 is provided below.

The endogeneity of leverage creates possible concern regarding the satisfaction of A1 and A3. First, due to agency costs, firms with good growth options can be expected to issue less debt. Coupling good growth options with low leverage leads to high investment. However, if the data reveal that such firms do invest more, it is precisely because they anticipated the ex post incentive compatible investment policy for equity, as given by (34). That is, causation works from leverage to investment, and not vice versa. Therefore, there is no a priori reason to assume that the endogenous choice of leverage causes violation of the stated independence assumptions.

A second more troubling concern is the issuance of debt in order to fund contemporaneous investment. This may lead to violation of A1. As a robustness check, we estimate all specifications using current R/K and then lagged R/K. The tradeoff here is that using lagged R/K cuts one year from the sample.

[13] See Greene (1997, p. 440) for a discussion.

Anticipating, results do not vary with the lag structure. As a final check, we conduct a GMM J-test of the overidentifying restrictions to detect violations of the stated assumptions needed to ensure consistency.

We now move on to discussion of the moment conditions exploited in the estimation process. Let

$$\begin{aligned} \mathbf{m}_i &\equiv [E(\mathbf{z}_j'\mathbf{z}_j)]^{-1}E[\mathbf{z}_j' i_j] \\ \mathbf{m}_x &\equiv [E(\mathbf{z}_j'\mathbf{z}_j)]^{-1}E[\mathbf{z}_j' x_j]. \end{aligned} \tag{39}$$

There are three second-order moment conditions, with four unknowns: $\beta, E(\omega_j^2)$, $E(u_j^2)$, and $E(\epsilon_j^2)$

$$\begin{aligned} E\left[(i_j - \mathbf{z}_j\mathbf{m}_i)^2\right] &= \beta^2 E\left[\omega_j^2\right] + E\left[u_j^2\right], \\ E\left[(i_j - \mathbf{z}_j\mathbf{m}_i)(x_j - \mathbf{z}_j\mathbf{m}_x)\right] &= \beta E\left[\omega_j^2\right], \\ E\left[(x_j - \mathbf{z}_j\mathbf{m}_x)^2\right] &= E\left[\omega_j^2\right] + E\left[\epsilon_j^2\right]. \end{aligned} \tag{40}$$

We have the following third-order moment conditions:

$$\begin{aligned} E\left[(i_j - \mathbf{z}_j\mathbf{m}_i)^2(x_j - \mathbf{z}_j\mathbf{m}_x)\right] &= \beta^2 E\left[\omega_j^3\right], \\ E\left[(i_j - \mathbf{z}_j\mathbf{m}_i)(x_j - \mathbf{z}_j\mathbf{m}_x)^2\right] &= \beta E\left[\omega_j^3\right]. \end{aligned} \tag{41}$$

The two additional moment conditions give us only one more unknown, $E[\omega_j^3]$. Under A4 and A5, (40) and (41) represent a system of five equations in five unknowns based on moment conditions up to the third-order. Proceeding in similar fashion, it is possible to show that using higher order moment conditions produces overidentified systems, offering alternative parameter estimates, and serving as the basis for overidentification tests.

This procedure produces a parameter estimate for each year of cross-sectional data. However, it is asymptotically more efficient to minimize a quadratic in the vector of unknown parameters, where the matrix of the quadratic form is the inverse of the asymptotic covariance matrix. This is referred to as the classical minimum distance estimator. Following Erickson and Whited (2000), we present classical minimum distance parameter estimates, beginning with those based on product moments up to the third-order (GMM3) and continuing up to those based on product moments up to the fifth-order (GMM5).

C. Data

In order to clarify the exact role played by the overhang correction, we use the Erickson and Whited (2000) data, which are drawn from a sample of 3,869 manufacturing firms in SIC's 2000-3999. Firms with missing data, potential coding errors, or mergers accounting for more than 15% of book value are deleted. This results in a balanced panel of 737 firms over the period 1992 to 1995. Since bond ratings are needed in order to impute the value of R/K and to sort firms into

Table I

Descriptive Statistics

The full sample consists of the 278 firms with bond ratings included in Erickson and Whited (2000) data during the period 1992 to 1995. Data are normalized by the estimated replacement value of the capital stock. Investment is total reported spending on plant, property, and equipment, excluding spending on acquisitions. Average Q is the estimated market value of the capital stock, exclusive of inventories, normalized by the replacement value of the capital stock. Details regarding the required imputations for computing Q are provided in Whited (1992). Cash flow is equal to income plus depreciation. The recovery claim is the imputed market value of total lender recoveries in default. Details on the imputation are provided in Appendix B.

Variable	25th Percentile	Median	75th Percentile	Average	Standard Deviation
Full Sample					
Normalized investment	0.060	0.100	0.150	0.120	0.100
Average Q	0.725	1.505	2.895	2.254	2.283
Normalized cash flow	0.080	0.120	0.170	0.136	0.080
Normalized recovery claim	0.004	0.008	0.026	0.029	0.063
Above Investment Grade					
Normalized investment	0.070	0.110	0.160	0.125	0.098
Average Q	0.840	1.750	3.115	2.478	2.386
Normalized cash flow	0.090	0.120	0.170	0.136	0.067
Normalized recovery claim	0.003	0.006	0.012	0.012	0.021
Below Investment Grade					
Normalized investment	0.035	0.070	0.120	0.096	0.094
Average Q	0.540	0.955	1.705	1.491	1.679
Normalized cash flow	0.070	0.115	0.170	0.134	0.114
Normalized recovery claim	0.027	0.049	0.096	0.088	0.108

high and low categories, we drop those firms for which ratings are not available for all 4 years, leaving us with a balanced panel of 278 firms. Details regarding the method used to compute average Q may be found in Whited (1992). The imputation of R/K is discussed in Appendix B.

Table I reports descriptive statistics. From the sample of 1,112 firm-year observations, 252 (22.7%) are below investment grade. Firms below investment grade exhibit much lower investment and Q values, and slightly lower cash flow. Of particular interest is the fact that the normalized recovery claim (R/K) is much larger for firms below investment grade. The median value of R/K is equal to 0.049 for firms below investment grade, but only 0.006 for firms above investment grade. The mean (median) ratio of R/K to average Q is 11% (6%) for firms below investment grade. By way of contrast, the mean (median) ratio of the R/K to average Q is only 0.8% (0.6%) for those above investment grade.

D. Estimation Results

The theory is tested using four variants of the vector of regressors listed in (35). Models 1 and 2 use bond ratings above investment grade as the basis

Table II

p-Values from Identification Tests

The null hypothesis is that the true Q coefficient is zero and/or that the residuals from a projection of true Q on the other regressors, excluding measured Q, are not skewed. The model is identified if the null is false. Model 1 uses the following regressors: Q; dummy for bond rating above investment grade; separate cash flow variables for firms above and below investment grade; R/K, which is the imputed market value of lenders' recovery claim in default normalized by the capital stock; and R/K interacted with the indicator for bond rating above investment grade. Model 2 is identical to Model 1, with the exception that the lagged value of R/K is used instead of the current value. Model 3 uses the following regressors: Q; dummy for recovery ratio at/or above sample median of 41.5%; separate cash flow variables for firms above and below investment grade; R/K; and R/K interacted with the indicator for recovery ratio above the sample median. Model 4 is identical to Model 3, with the exception that the lagged value of R/K is used instead of the current value. The sample consists of the 278 firms with bond ratings included in Erickson and Whited (2000) data during the period 1992 to 1995.

	1992	1993	1994	1995
Model 1	0.077	0.066	0.090	0.035
Model 2	NA	0.061	0.091	0.031
Model 3	0.039	0.087	0.127	0.028
Model 4	NA	0.078	0.125	0.026

for categorizing a firm-year as having *HIGH* status. The difference between Models 1 and 2 is that the former uses contemporaneous R/K, while the latter uses lagged R/K as a robustness check. Models 3 and 4 use industry-specific recovery ratios as the basis for categorizing a firm as having *HIGH* status, with the former using contemporaneous R/K and the latter using lagged R/K as a robustness check.

Recall that the models are identified only if the coefficient on true Q is nonzero and the residuals from a projection of true Q on the vector of other regressors are skewed. Table II reports p-values from identification tests, where the null hypothesis is that $\beta = 0$ and/or $E[\omega_j^3] = 0$. There is reasonable basis for rejecting the null for all model variants. For Models 1 and 2, the null is rejected at the 10% level in each year. The case for identification is somewhat weaker for Models 3 and 4, with the p-values below 10% for all years other than 1994, where the p-values are just under 13%.

Table III reports parameter estimates for Model 1. For the purpose of contrast, we report parameter estimates from OLS in addition to those from GMM3 to GMM5. Consistent with the results in Erickson and Whited (2000), the Q-coefficients generated using GMM are more than three times the magnitude of the point estimates from OLS. The existence of debt overhang is supported by the negative coefficient on R/K, which is always statistically significant. Erickson and Whited document that t-tests on perfectly measured regressors tend to over-reject in finite samples. The high degree of significance suggests that this is not an important concern. The coefficient on R/K is also economically significant. To see this, note that if $\Lambda_L = 0$, the structural model indicates that the coefficients on average Q and R/K are equal to η^{-1} and $-\eta^{-1}$,

Table III

Test of Overhang Effect by Bond Rating Using the Current Value of the Normalized Recovery Claim (Model 1)

The dependent variable is capital expenditures normalized by the capital stock. The explanatory variables are: Q; dummy for bond rating above investment grade; separate cash flow variables for firms above and below investment grade; R/K, which is the the imputed market value of lenders' recovery claim in default normalized by the capital stock; and R/K interacted with the indicator for bond rating above investment grade. We report minimum distance estimates corresponding to the four types of cross-section estimators: OLS, GMM3, GMM4, and GMM5. The sample consists of the 278 firms with bond ratings included in Erickson and Whited (2000) data during the period 1992 to 1995. Standard errors are reported in parentheses. For OLS, White standard errors are reported.

	OLS	GMM3	GMM4	GMM5
Intercept	0.025***	0.030***	0.031***	0.031***
	(0.008)	(0.008)	(0.008)	(0.008)
Average Q	0.013***	0.041***	0.037***	0.052***
	(0.003)	(0.011)	(0.002)	(0.003)
DUM = 1 if rating above BB+	0.007	−0.005	0.004	0.003
	(0.013)	(0.018)	(0.015)	(0.015)
R/K	−0.174***	−0.312***	−0.125**	−0.173***
	(0.053)	(0.094)	(0.052)	(0.059)
DUM ∗ (R/K)	−0.200	0.039	−0.170	−0.090
	(0.214)	(0.399)	(0.197)	(0.212)
Cash flow: Rating at/below BB+	0.441***	−0.004	0.251***	0.208***
	(0.040)	(0.125)	(0.043)	(0.047)
Cash flow: Rating above BB+	0.483***	−0.123	0.054	−0.152
	(0.094)	(0.253)	(0.131)	(0.144)
R^2	0.391	0.642	0.651	0.602

The symbols ***, **, and * indicate statistical significance at the 1, 5, and 10% levels, respectively.

respectively. The estimated coefficients on R/K are well in excess of typical estimates of the Q coefficient. The insignificant coefficients on the dummy variable for *HIGH* status and the interaction term between *HIGH* and R/K indicate that issuance of additional secured debt does not mitigate the overhang effect for firms with high debt ratings.

Table IV contains parameter estimates from Model 2, where lagged R/K is used as a regressor in order to deal with potential endogeneity concerns. The parameter estimates from Model 2 are similar to those from Model 1, with the coefficients on Q and R/K always significant at the 1% level. Once again, we find no evidence that the overhang effect is mitigated for higher-rated firms, with the indicator for *HIGH* status and the interaction term insignificant across all specifications.

The parameter estimates from Models 3 and 4, which use recovery ratios as the basis for assigning firms *HIGH* status, are contained in Tables V and VI. The results are similar to those for Models 1 and 2. Average Q and R/K are always significant at the 1% level. The insignificance of the *HIGH* dummy variable

Table IV

Test of Overhang Effect by Bond Rating Using the Lagged Value of the Normalized Recovery Claim (Model 2)

The dependent variable is capital expenditures normalized by the capital stock. The explanatory variables are: Q; dummy for bond rating above investment grade; separate cash flow variables for firms above and below investment grade; lagged R/K, which is the imputed market value of lenders' recovery claim in default normalized by the capital stock; and lagged R/K interacted with the indicator for bond rating above investment grade. We report minimum distance estimates corresponding to the four types of cross-section estimators: OLS, GMM3, GMM4, and GMM5. The sample consists of the 278 firms with bond ratings included in Erickson and Whited (2000) data during the period 1993 to 1995. Standard errors are reported in parentheses. For OLS, White standard errors are reported.

	OLS	GMM3	GMM4	GMM5
Intercept	0.019**	0.033***	0.030***	0.030***
	(0.009)	(0.009)	(0.008)	(0.008)
Average Q	0.012***	0.043***	0.063***	0.066***
	(0.003)	(0.010)	(0.004)	(0.004)
DUM = 1 if rating above BB+	0.006	−0.008	0.006	0.005
	(0.014)	(0.018)	(0.016)	(0.016)
Lagged (R/K)	−0.220***	−0.333***	−0.215***	−0.272***
	(0.068)	(0.096)	(0.074)	(0.082)
DUM ∗ Lagged (R/K)	−0.011	0.120	−0.085	−0.089
	(0.225)	(0.443)	(0.264)	(0.309)
Cash flow: Rating at/below BB+	0.520***	−0.059	0.142**	0.143∗
	(0.046)	(0.112)	(0.068)	(0.073)
Cash flow: Rating above BB+	0.477***	−0.162	−0.009	−0.270
	(0.100)	(0.251)	(0.156)	(0.179)
R^2	0.408	0.627	0.636	0.591

The symbols ***, **, and * indicate statistical significance at the 1, 5, and 10% levels, respectively.

and the interaction term indicate that there is no evidence for mitigation of the overhang effect for firms expected to have superior ability in issuing additional secured debt.

Table VII reports p-values for the J-statistics of the alternative models. The results are consistent with the overidentifying restrictions, given that we fail to reject at the 5% level, while finding only three rejections at the 10% level.

Taken together, these results provide strong evidence in favor of the existence of debt overhang and against the notion that firms utilize additional secured debt issuance as a device for mitigating the problem. Although this second finding runs counter to standard theory, a number of potential explanations are possible. First, incremental investment may take the form of assets that are inherently difficult to pledge as collateral. Second, existing covenant protections may be sufficient to give senior lenders inviolable collateral. Finally, even in the absence of binding credit constraints, firms may be reluctant to expropriate existing lenders due to reputational concerns.

The data suggest a potential puzzle, however. Note that in the absence of a collateral channel, the basic structural model implies that coefficients on average

Table V

Test of Overhang Effect by Recovery Ratio Using the Current Value of the Normalized Recovery Claim (Model 3)

The dependent variable is capital expenditures normalized by the capital stock. The explanatory variables are: Q; dummy for recovery ratio at/above the sample median of 41.5%; separate cash flow variables for firms above and below investment grade; R/K which is the imputed market value of lenders' recovery claim in default normalized by the capital stock; and R/K interacted with the indicator for recovery ratio at/above the sample median. We report minimum distance estimates corresponding to the four types of cross-section estimators: OLS, GMM3, GMM4, and GMM5. The sample consists of the 278 firms with bond ratings included in the Erickson and Whited (2000) data during the period 1992 to 1995. Standard errors are reported in parentheses. For OLS, White standard errors are reported.

	OLS	GMM3	GMM4	GMM5
Intercept	0.028***	0.030***	0.031***	0.030***
	(0.007)	(0.011)	(0.009)	(0.009)
Average Q	0.013***	0.047***	0.042***	0.038***
	(0.003)	(0.011)	(0.003)	(0.004)
DUM = 1 if recovery ratio $\geq$ 41.5%	0.009	−0.001	0.004	0.005
	(0.008)	(0.011)	(0.009)	(0.010)
R/K	−0.226***	−0.318***	−0.190***	−0.185***
	(0.068)	(0.077)	(0.063)	(0.065)
DUM $*$ (R/K)	0.028	−0.052	−0.070	−0.140
	(0.080)	(0.114)	(0.085)	(0.093)
Cash flow: Rating at/below BB+	0.417***	0.012	0.272***	0.236***
	(0.047)	(0.148)	(0.061)	(0.065)
Cash flow: Rating above BB+	0.445***	−0.251	−0.074	−0.121
	(0.074)	(0.228)	(0.097)	(0.111)
R^2	0.394	0.654	0.641	0.604

The symbols ***, **, and * indicate statistical significance at the 1, 5, and 10% levels, respectively.

Q and R/K are equal to η^{-1} and $-\eta^{-1}$, respectively. However, our point estimates of the coefficients on R/K are roughly 3–5 times that on Q. A reasonable interpretation of this finding is that the negative effect of debt on investment is not limited to the overhang channel. For instance, Asquith, Gertner, and Scharfstein (1994) find that firms encountering distress may cut investment drastically in an attempt to maintain sufficient liquidity to avoid default. Similarly, it is possible that healthy firms respond to rating downgrades by hoarding cash and avoiding discretionary capital expenditures. Formal tests of this channel, and others by which debt may reduce investment, offer an interesting area for future research.

IV. Conclusions

As the stock of theoretical models increases, the ability of corporate finance economists to determine their significance will hinge upon our ability to spell out precise testable hypotheses. Incorporating debt in a dynamic capital

Table VI

Test of Overhang Effect by Recovery Ratio Using the Lagged Value of the Normalized Recovery Claim (Model 4)

The dependent variable is capital expenditures normalized by the capital stock. The explanatory variables are: Q; dummy for recovery ratio at/above the sample median of 41.5%; separate cash flow variables for firms above and below investment grade; lagged R/K, which is the the imputed market value of lenders' recovery claim in default normalized by the capital stock; and lagged R/K interacted with the indicator for recovery ratio at/above the sample median. We report minimum distance estimates corresponding to the four types of cross-section estimators: OLS, GMM3, GMM4, and GMM5. The sample consists of the 278 firms with bond ratings included in Erickson and Whited (2000) data during the period 1993 to 1995. Standard errors are reported in parentheses. For OLS, White standard errors are reported.

	OLS	GMM3	GMM4	GMM5
Intercept	0.030***	0.030***	0.033***	0.031***
	(0.007)	(0.011)	(0.009)	(0.009)
Average Q	0.012***	0.047***	0.060***	0.061***
	(0.003)	(0.010)	(0.004)	(0.006)
DUM = 1 if recovery ratio $\geq$ 41.5%	0.005	0.002	0.000	0.000
	(0.008)	(0.011)	(0.009)	(0.010)
Lagged (R/K)	−0.248***	−0.289***	−0.276***	−0.293***
	(0.060)	(0.071)	(0.063)	(0.069)
DUM $*$ Lagged (R/K)	0.184**	−0.009	0.132	0.065
	(0.083)	(0.145)	(0.104)	(0.133)
Cash flow: Rating at/below BB+	0.453***	−0.005	0.263***	0.209**
	(0.052)	(0.137)	(0.073)	(0.099)
Cash flow: Rating above BB+	0.477***	−0.258	−0.172	−0.398***
	(0.079)	(0.225)	(0.113)	(0.153)
R^2	0.411	0.641	0.624	0.599

The symbols ***, **, and * indicate statistical significance at the 1, 5, and 10% levels, respectively.

accumulation model, this paper derives an empirical proxy for levered equity's marginal Q, generating a direct test for debt overhang and mitigation of the effect through issuance of additional secured debt. In the empirical section, we tested the structural model finding that overhang is significant, with no evidence that the problem is mitigated for firms with superior ability to issue additional secured debt. In fact, our estimates suggest that the negative effect of debt on investment is stronger than would be implied by the debt overhang channel working in isolation.

The empirical analysis presented in this paper is necessarily limited to testing a subset of the testable hypotheses generated by the model, leaving open for future research the following predictions regarding the relationship between asset life and endogenous leverage and investment determination. First, within a given firm, debt overhang distorts investment composition and not simply its level. In particular, overhang creates a greater relative bias against investments in long-lived assets, such as land and physical plant, as opposed to short-lived assets such as desktop computers. Second, the bias against

Table VII

p-Values for J-Tests of Overidentifying Restrictions

The null hypothesis is that the overidentifying restrictions are valid. The abbreviation GMM4 refers to the generalized method of moments estimator based on moments up to the fourth order, while GMM5 exploits moments up to the fifth order. Model 1 uses the following regressors: Q; dummy for bond rating above investment grade; separate cash flow variables for firms above and below investment grade; R/K, which is the imputed market value of lenders' recovery claim in default normalized by the capital stock; and R/K interacted with the indicator for bond rating above investment grade. Model 2 is identical to Model 1, with the exception that the lagged value of R/K is used instead of the current value. Model 3 uses the following regressors: Q; dummy for recovery ratio at/or above sample median of 41.5%; separate cash flow variables for firms above and below investment grade; R/K; and R/K interacted with the indicator for recovery ratio above the sample median. Model 4 is identical to Model 3, with the exception that the lagged value of R/K is used instead of the current value. The sample consists of the 278 firms with bond ratings included in Erickson and Whited (2000) data during the period 1992 to 1995.

	Model 1	Model 2	Model 3	Model 4
		GMM4		
1992	0.484	NA	0.444	NA
1993	0.512	0.505	0.523	0.526
1994	0.185	0.204	0.209	0.214
1995	0.096	0.115	0.112	0.121
		GMM5		
1992	0.652	NA	0.651	NA
1993	0.684	0.691	0.690	0.665
1994	0.073	0.063	0.116	0.122
1995	0.117	0.170	0.222	0.226

particular asset classes is mitigated the greater their collateral value with respect to incremental debt finance. Finally, endogenous leverage determination depends on the asset life of growth options held by the firm, and not simply their stand-alone value. This casts doubt on the practice of treating average Q as a sufficient statistic for the effect of growth options on optimal leverage.

Appendix A

We derive all the results for the extended model with new secured debt, since the results for the basic model follow by setting $\lambda = 0$. For simplicity, we introduce the infinitesimal generator A. For an arbitrary C^2 function h of (K, ε), Ito's lemma implies:

$$E[dh]\frac{1}{dt} \equiv A(h) = (I - \delta K)h_K + \mu(\varepsilon)h_\varepsilon + \frac{1}{2}\sigma^2(\varepsilon)h_{\varepsilon\varepsilon}.$$

Proof of Propositions 2 and 4: The Bellman equation is

$$rS(K, \varepsilon) = \max_I \ \pi(K, \varepsilon) - c(I, K) + \lambda I - b + A(S). \tag{A1}$$

The Bellman equation holds identically in K for all (K, ε), implying that the derivatives with respect to K of the left and right sides of (A1) must match. Differentiating with respect to K yields

$$rS_K = \pi_K - c_K - \delta S_K + (I - \delta K)S_{KK} + \mu(\varepsilon)S_{K\varepsilon} + \tfrac{1}{2}\sigma^2(\varepsilon)S_{K\varepsilon\varepsilon}. \quad \text{(A2)}$$

Using the generator A this may be rewritten as

$$rq = \pi_K - c_K + A(q) - \delta q. \quad \text{(A3)}$$

Multiplying by K and using the fact that $KA(q) - \delta qK = A(qK) - qI$, we have

$$rqK = \pi_K K - c_K K + A(qK) - qI. \quad \text{(A4)}$$

Using the optimality condition $(q + \lambda)I = c_I I$, and homogeneity of π and c yields

$$rqK = \pi - c + \lambda I + A(qK). \quad \text{(A5)}$$

Subtracting (A5) from (A1) and rearranging terms yields

$$A(S - qK) - r(S - qK) = b. \quad \text{(A6)}$$

Now define the Ito process ξ as

$$\xi(K, \varepsilon, t) \equiv e^{-rt}[S(K_t, \varepsilon_t) - K_t q(K_t, \varepsilon_t)]. \quad \text{(A7)}$$

Application of Ito's lemma to ξ yields

$$d\xi_t = e^{-rt}[A(S - qK) - r(S - qK)]\,dt + e^{-rt}\sigma(\varepsilon_t)(S_\varepsilon - Kq_\varepsilon)\,dW_t. \quad \text{(A8)}$$

Substituting in (A6), we may rewrite $d\xi$ as

$$d\xi_t = e^{-rt}b\,dt + e^{-rt}\sigma(\varepsilon_t)(S_\varepsilon - Kq_\varepsilon)\,dW_t. \quad \text{(A9)}$$

Integrating ξ up to the optimal default date T and taking expectations, we have

$$\begin{aligned} &E\left[e^{-rT}(S(K_T, \varepsilon_T) - K_T q(K_T, \varepsilon_T))\right] \\ &= S(K_0, \varepsilon_0) - K_0 q(K_0, \varepsilon_0) + E\left[\int_0^T e^{-rt}b\,dt\right] \\ &\quad + E\left[\int_0^T e^{-rt}\sigma(\varepsilon_t)[S_\varepsilon(K_t, \varepsilon_t) - K_t q_\varepsilon(K_t, \varepsilon_t)]\,dW_t\right]. \end{aligned} \quad \text{(A10)}$$

Substituting in the value matching and smooth pasting conditions, $S(K_T, \varepsilon_T) = 0$ and $q(K_T, \varepsilon_T) = 0$, and noting that the last term in (A10) is a martingale with expectation zero, (A10) simplifies to

$$K_0 q(K_0, \varepsilon_0) = S(K_0, \varepsilon_0) + E\left[\int_0^T e^{-rt}b\,dt\right]. \quad \text{(A11)}$$

The last step is to derive the senior debt price, which carries a default claim on existing capital plus the fraction of future investment not pledged to new lenders. Therefore,

$$D(K_0, \varepsilon_0) = E\left[\int_0^T e^{-rt} b\, dt\right] + (1-\alpha) \times E\left[e^{-rT} v(\varepsilon_T)\left(e^{-\delta T} K_0 + \int_0^T (1-\gamma_t) e^{-\delta(T-t)} I_t\, dt\right)\right]. \quad \text{(A12)}$$

From Lemma 1 we know that under the stated homogeneity assumptions

$$R(K_0, \varepsilon_0) = (1-\alpha) E\left[e^{-rT} v(\varepsilon_T) K_T\right]. \quad \text{(A13)}$$

Additionally, the terminal capital stock is

$$K_T = e^{-\delta T} K_0 + \int_0^T e^{-\delta(T-t)} I_t\, dt. \quad \text{(A14)}$$

From (A13) and (A14) it follows that (A12) may be restated as

$$D(K_0, \varepsilon_0) = E\left[\int_0^T e^{-rt} b\, dt\right] + R(K_0, \varepsilon_0) - E\left[\int_0^T e^{-rt}[(1-\alpha)\gamma_t e^{-(r+\delta)(T-t)} v(\varepsilon_T)] I_t\, dt\right]. \quad \text{(A15)}$$

Since secured debt carries price λ, we know $\forall t < T$

$$\lambda I_t\, dt = (1-\alpha)\gamma_t E_t[e^{-(r+\delta)(T-t)} v(\varepsilon_T)] I_t\, dt. \quad \text{(A16)}$$

From (A16), it follows that

$$D(K_0, \varepsilon_0) = E\left[\int_0^T e^{-rt} b\, dt\right] + R(K_0, \varepsilon_0) - \lambda E\left[\int_0^T e^{-rt} I_t\, dt\right]. \quad \text{(A17)}$$

Substituting (A17) into (A11) yields the result. Q.E.D.

Proof of Propositions 1 and 3: Rearranging (A3) yields

$$A(q) - (r+\delta) q = -[\pi_K - c_K]. \quad \text{(A18)}$$

Redefine the Ito process ξ as $\xi(K, \varepsilon, t) \equiv e^{-(r+\delta)t} q(K_t, \varepsilon_t)$ and repeat the procedures in equations (A8)–(A11). Q.E.D.

Appendix B

Collections in the event of default are computed as long-term debt multiplied by industry specific recovery ratios (*rr*). Recovery ratios by three-digit SIC code are from Altman and Kishore (1996), who compute averages from 1971 to 1995. Collections are multiplied by the value of a hitting claim paying one dollar at default. Default probabilities (ϕ_{jt}) by bond rating over a 20-year horizon are from Moody's. Since the hitting claim could have a low or negative beta, discount factors (d_t) are based on long-term Treasuries. The hitting claim value is therefore

$$\sum_{t=1}^{20} \phi_{jt}\, d_t. \tag{B1}$$

Letting LTD denote the value of long-term debt, the imputation for R/K is simply the product of recovery ratio, leverage, and the value of the hitting claim:

$$\widehat{\left(\frac{R}{K}\right)} = rr_j * \left(\frac{LTD}{K}\right) * \left(\sum_{t=1}^{20} \phi_{jt} d_t\right). \tag{B2}$$

REFERENCES

Abel, Andrew, 1983, Optimal investment under uncertainty, *American Economic Review* 73, 228–233.

Abel, Andrew, and Janice Eberly, 1994, A unified model of investment under uncertainty, *American Economic Review* 84, 1369–1384.

Abel, Andrew, and Janice Eberly, 2001, Investment and q with fixed costs: An empirical analysis, Working paper, University of Pennsylvania.

Altman, Edward, and Vellore Kishore, 1996, Almost everything you wanted to know about recoveries on defaulted bonds, *Financial Analysts Journal*, November/December, 57–64.

Asquith, Paul, Robert Gertner, and David Scharfstein, 1994, Anatomy of financial distress: An examination of junk-bond issuers, *Quarterly Journal of Economics* 109, 625–658.

Barnett, Steven, and Plutarchos Sakellaris, 1998, Nonlinear response of firm investment to q: Testing a model of convex and non-convex adjustment costs, *Journal of Monetary Economics* 42, 261–288.

Caballero, Ricardo, 1997, Aggregate investment, in J. Taylor, and M. Woodford, eds. *Handbook of Macroeconomics* (North-Holland, Amsterdam).

Chirinko, Robert, 1987, Tobin's q and financial policy, *Journal of Monetary Economics* 19, 69–87.

Duffie, Darrell, 1996, *Dynamic Asset Pricing Theory*, 2nd ed. (Princeton University Press, Princeton, NJ).

Dumas, Bernard, 1991, Super contact and related optimality conditions, *Journal of Economic Dynamics and Control* 15, 675–695.

Erickson, Timothy, and Toni Whited, 2000, Measurement error and the relationship between investment and q, *Journal of Political Economy* 108, 1027–1057.

Erickson, Timothy, and Toni Whited, 2001, On the information content of different measures of q, Working paper, University of Iowa.

Erickson, Timothy, and Toni Whited, 2002, Two-step GMM estimation of the errors in variables model using high order moments, *Econometric Theory* 18, 776–799.

Fleming, Wendell, and H. Mete Soner, 1993, *Controlled Markov Processes and Viscosity Solutions* (Springer-Verlag, New York).

Greene, William, 1997, *Econometric Analysis*, 3rd ed. (Prentice Hall, Upper Saddle River, NJ).

Hayashi, Fumio, 1982, Tobin's marginal q and average q: A neoclassical interpretation, *Econometrica* 50, 213–224.

Jensen, Michael, 1986, Agency costs of free cash flow, corporate finance, and takeovers, *American Economic Review* 76, 323–329.

Jensen, Michael, and William Meckling, 1976, Theory of the firm: Managerial behavior, agency costs, and ownership structure, *Journal of Financial Economics* 3, 305–360.

Karatzas, Ioannis, and Steven Shreve, 1988, *Brownian Motion and Stochastic Calculus* (Springer-Verlag, New York).

Lang, Lawrence, Eli Ofek, and Rene Stulz, 1996, Leverage, investment, and firm growth, *Journal of Financial Economics* 40, 3–29.

Leland, Hayne, 1994, Corporate debt value, bond covenants, and optimal capital structure, *Journal of Finance* 49, 1213–1252.

Mauer, David, and Alexander Triantis, 1994, Interactions of corporate financing and investment decisions: A dynamic framework, *Journal of Finance* 49, 1253–1277.

Mello, Antonio, and John Parsons, 1992, Measuring the agency costs of debt, *Journal of Finance* 47, 1887–1904.

Morellec, Erwan, 2001, Asset liquidity, capital structure, and secured debt, *Journal of Financial Economics* 61, 173–206.

Moyen, Nathalie, 2001, How big is the debt overhang problem? Working paper, University of Colorado at Boulder.

Mussa, Michael, 1977, External and internal adjustment costs and the theory of aggregate and firm investment, *Econometrica* 44, 163–178.

Myers, Stewart, 1977, Determinants of corporate borrowing, *Journal of Financial Economics* 5, 147–175.

Parrino, Robert, and Michael Weisbach, 1999, Measuring investment distortions arising from stockholder–bondholder conflicts, *Journal of Financial Economics* 53, 3–42.

Poterba, James, and Lawrence Summers, 1983, Dividend taxes, corporate investment, and q, *Journal of Public Economics* 22, 135–167.

Smith, Clifford, and Jerold Warner, 1979, On financial contracting: An analysis of bond covenants, *Journal of Financial Economics* 7, 117–161.

Summers, Lawrence, 1981, Taxation and corporate investment: A q-theory approach, *Brookings Papers on Economic Activity* 1, 67–127.

Titman, Sheridan, and Sergei Tsyplakov, 2002, A dynamic model of optimal capital structure, Working paper, University of Texas at Austin.

Whited, Toni, 1992, Debt, liquidity constraints, and corporate investment: Evidence from panel data, *Journal of Finance* 47, 1425–1460.